Cherednik operators and Hecke algebras

Let $\mathfrak{a}$ be a real Euclidean vector space and let Σ be a root system in $\mathfrak{a}$. A multiplicity function is a complex-valued function m on Σ which is invariant with respect to the Weyl group of Σ. In the mid 1990s, Ivan Cherednik associated with a triple $(\mathfrak{a}, \Sigma, m)$ a commutative family of first order differential-reflection operators, nowadays known as Cherednik operators or trigonometric Dunkl operators. The original motivation for the study of these operators came from the theory of invariant differential operators: if the triple $(\mathfrak{a}, \Sigma, m)$ arises from the structure theory of a Riemannian symmetric space of the non-compact type G/K, then it is possible to explicitly construct all radial components of the G-invariant differential operators on G/K using the Cherednik operators. The joint spectral theory of Cherednik operators is therefore naturally related to the harmonic analysis on Riemannian symmetric spaces (and to the more general theory of hypergeometric functions in several variables of Heckman and Opdam). But it is also related with the representation theory of the graded Hecke algebra of Lusztig. The goal of this course is to introduce the Cherednik operators and study these relations.

Outline of the course:

(1) Preliminaries on root systems and their Weyl groups
(2) Cherednik operators
(3) The graded Hecke algebra and its finite-dimensional representations.
(4) Polynomial eigenfunctions of the Cherednik operators; Jacobi polynomials in several variables.
(5) Construction of the radial components of invariant differential operators for Riemannian symmetric spaces of noncompact type.
(6) Invariant hermitian structures for the graded Hecke algebra
(7) Compact harmonic analysis and applications
(8) Symmetric and non-symmetric hypergeometric functions associated with root systems

There are many references on the subject of this course. Our starting point will be the following two articles:

E. M. Opdam. Harmonic analysis for certain representations of graded Hecke algebras, Acta Mathematica, 175 (1995), 75-121.

E. M. Opdam. Lectures on Dunkl operators. In: MSJ Memoires, Part I, volume 8, Mathematical Society of Japan, 2000.